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Dynamical Determination of Dilaton and Moduli Vacuum Expectation Values

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Abstract

We determine the dilaton and moduli vacuum expectation values using the one-loop effective potential and topological constraints. A new ingredient of this analysis is that we use a dilaton Kähler potential that includes renormalization effects to all loops. We find that the dilaton vacuum expectation value is related to certain topological properties of the compact spacetime. We demonstrate that values of the dilaton vacuum expectation value that are consistent with the weak scale measurements can be dynamically obtained in this fashion.

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The determination of the dilaton (S) and moduli vacuum expectation values (VEV's) is an important problem in string phenomenology because it is directly related to the predictions of the string models [1]. The dilaton and moduli fields are flat directions to all orders in string perturbation theory, and it is hoped that some non-perturbative effects will lift these flat directions and determine these VEV's dynamically.

Non-perturbative gaugino condensation in the hidden sector is a ready candidate for specifying the dilaton and moduli VEV's. The hidden sector of the heterotic string is asymptotically free and will "confine" at a low energy and give rise to a dilaton-dependent superpotential. However, it is found that this superpotential generically leads to a "runaway" dilaton VEV, *i.e.* the potential energy is minimized at either $S \to 0$ or $S \to \infty$.

So far, there are two types of plausible string-inspired gaugino condensation scenarios. One is the c-number scheme [2,3] in which it is assumed that a constant term is induced in the superpotential by the gaugino condensation to cancel the dilaton dependent contribution. The other is multiple gaugino condensation [4,5,6,7], in which multiple gauge groups condense in the hidden sector.

In this work, we discuss the determination of the dilaton and moduli VEV's through the one-loop effective potential of the gaugino condensation dynamics in the hidden sector. We will discuss particularly the c-number gaugino condensation models proposed in [3] for string models with no-scale structure and in which the gauge coupling constant does not receive string threshold corrections. The no-scale structure plays the crucial role in generating the large mass hierarchy in these models. Because of the no-scale structure, the dialton and moduli VEVs are not determined at the tree level and the examination of the one-loop effective potential becomes necessary.

The other purpose of this paper is to examine whether the inclusion of higher-loop corrections to the dilaton Kähler potential improves the dilaton runaway problem. In ref. [9], we pointed out that the one-loop corrections to the dilaton Kähler potential may play a significant role in the gaugino condensation dynamics and in solving the dilaton runaway problem. Because the gauge coupling constant is closely related to the dilaton Kähler potential, and the gauge coupling is large at the gaugino condensation scale, the inclusion of higher loop correction becomes very important, which is partly the subject of this work.

The new ingredient of the present work is that we use a dilaton Kähler potential derived in Ref. [8], in which renormalization effects to all loops are taken into account. In our approach, both the effective potential and the quantization condition imposed by the compact manifold geometry or topology play important roles in determining the dilaton and moduli VEV's.

In Ref. [8] the E_8 gauge coupling and the modified dilaton–axion Kähler potential are derived to all orders for effective field theories arising from 4-dimensional $N = 1$ heterotic string models in which the gauge coupling constant does not receive string threshold corrections. In this paper, we will use the formulation of Ref. [8] and analyze its one-loop effective potential using the same methods as in Ref. [9].

We first describe the model we will analyze. The Kähler potential given in Ref [8] is

$$K = \log \left[\frac{1}{2} g^2 \left(1 + \frac{b_0}{3} g^2 \right)^{-3} \right]. \tag{1}$$

Here g^2 is the gauge coupling constant, which is related to the dilaton and the effective

gaugino bilinear field H through

$$g^{-2} = \frac{1}{2}(S+\bar{S}) - \frac{2b_0}{3}\log g^2 + b_0 \log |H|^2 + \frac{1}{2}a_0, \tag{2}$$

$$a_0 \equiv b_0\left(-2 + \frac{10}{3}\log 2\right). \tag{3}$$

We assume the usual superpotential obtained by symmetry arguments

$$W = d\left[\frac{1}{4}SH^3 + \frac{b_0}{2}H^3\log(\eta H)\right] + c_0 + W_0. \tag{4}$$

Here c_0 and W_0 are the contributions from the charged background VEV's and matter fields, respectively. The parameters d and η are not fixed by symmetry requirements, but are specified by the underlying gaugino condensation dynamics. In Ref [10,11] it was shown that $d = 1$, and we will use this result in the analysis below. We follow the argument in Ref [9] and treat the H field as a dynamical field. Additional details of the calculation is given in Refs. [12,9]

In the above model, the inverse of Kähler matric is

$$(K^{-1})^{i\bar{j}} = \begin{pmatrix} 1/x_2 + 4b_0^2C/(3|H|^2) & -2b_0C/(3H) & -\frac{2}{3}b_0C \\ -2b_0C/3\bar{H} & \frac{1}{3}C & \frac{1}{3}HC \\ -\frac{2}{3}b_0C & \frac{1}{3}\bar{H}C & \frac{1}{3}C(|H|^2+C) \end{pmatrix}, \tag{5}$$

where

$$C \equiv T + \bar{T} - |H|^2, \tag{6}$$

and

$$x_2 \equiv \frac{dx_1}{dy}\bigg/\frac{df}{dy} = \frac{y}{4(y+b_0/3)^2(y-2b_0/3)}, \tag{7}$$

$$x_1 \equiv \frac{dL}{dy}\bigg/\frac{df}{dy} = -\frac{1}{2(y+b_0/3)}, \tag{8}$$

$$\begin{aligned} L &\equiv \frac{1}{2}g^2\left(1+\frac{b_0}{3}g^2\right)^{-3} \\ &= \frac{1}{2}y^2\left(y+\frac{b_0}{3}\right)^{-3}, \end{aligned} \tag{9}$$

$$\begin{aligned} f &\equiv 2y + \frac{4b_0}{3}\log y + \frac{1}{2}c_0 \\ &= S + \bar{S}b_0 \log |H|^2, \end{aligned} \tag{10}$$

$$y \equiv 1/g^2. \tag{11}$$

Here g^2 is the gauge coupling constant at the gaugino condensation scale, which is determined by the parameter η through the eq.(2). In the following analysis, we will express everything in terms of g^2 instead of η.

Solving the tree-level vacuum condition

$$\langle\tilde{W}_H\rangle \equiv \left\langle W_H - \frac{L_H}{L}W\right\rangle = 0, \tag{12}$$

$$\langle\tilde{W}_S\rangle \equiv \langle]W_S + K_S W\rangle = 0, \tag{13}$$

we obtain

$$H = \frac{1}{\eta}e^{-S/(2b_0)}, \tag{14}$$

$$\langle W\rangle = c_0 = \frac{d}{4x_1}\langle H^3\rangle. \tag{15}$$

Using the vacuum conditions eqs. (12) and (13) and assuming that $S = \bar{S}, H = \bar{H}$ (*i.e.* CP violation is highly suppressed), we obtain the normalized fermion mass eigenvalues

$$m_{1,2}^{1/2} = e^G\left[\frac{1}{2}(1-x) \pm \sqrt{\frac{1}{4}(1-x)^2 + 3xz}\right], \qquad m_3^{1/2} = 0. \tag{16}$$

The scalar masses are

$$M_{1,2}^{s2} = e^G\left[\frac{1}{2}(2-x)^2 + 3xz \pm \frac{1}{2}(2-x)\sqrt{(2-x)^2 + 3xz}\right], \tag{17}$$

$$M_{3,4}^{s2} = e^G\left[\frac{1}{2}x^2 + 3xz \pm \frac{1}{2}x\sqrt{(x^2 + 3xz}\right], \tag{18}$$

$$M_{5,6}^{s2} = 0. \tag{19}$$

The gravitino mass is

$$M_{3/2} = e^G = \frac{1}{8}y^2(y + \frac{b_0}{3})^{-1}z^{-3}. \tag{20}$$

In the above

$$x \equiv \frac{x_1^2}{x_2} = 1 - \frac{2}{3}b_0 g^2, \qquad z \equiv C/|H|^2. \tag{21}$$

One can see that in the case of [9], $x = 1$, the above computation yields the same result as ref [9].

As in the one-loop case [9], the above result indicates that dilaton mass is on the same order as the H mass (to be identified with the scale of gaugino condensation) *independently of the value of the dilaton VEV*. Supersymmetry is broken in the hidden sector. Furthermore, the all-loop effect split the degeneracies between the dilaton and the H field mass in the one-loop case.

Next we examine the vacuum structure using the one-loop effective potential. The potential energy only depends on the modular invariant function $z = C/|H|^2$, and therefore the moduli and dilaton VEV's are not uniquely determined by the effective potential in this model. We take the cut-off scale to be

$$\Lambda^2 = |H|^2 e^{K/3}. \tag{22}$$

The one-loop effective potential is

$$v_1 = 64\pi^2 V^{1-\mathrm{loop}} = 2\,\mathrm{Str}(M^2\Lambda^2) + \mathrm{Str}\left[M^4 \log(M^2/\Lambda^2)\right]. \tag{23}$$

Given a value for y, we can minimize the effective potential with respect to z. For $y \sim 1$ we find a global minimum at $z \sim 1$. For example, for E_8 hidden sector $b_0 = 90/16\pi^2$, $z = 0.63$ for $y = 1$, $z = 0.36$ for $y = 0.1$. For the E_6 hidden sector, $b_0 = 36/16\pi^2$, $z = 0.73$ for $y = 1$ and $z = 0.22$ for $y = 0.1$.

The above result does not yield the hierarchy between the moduli VEV and the gaugino condensation scale. Therefore, the proposal in [9] that the inclusion of the higher loop correction to the dilaton Kähler potential might generate a large z does not work here. In fact, one can see this immediately from the Kähler potential in [8]. It is proposed in [9] that the dilaton Kähler potential getting large at the gaugino condensation scale may lead to the large z. However, the all-loop corrected dilaton Kähler potential in [8] does not become large as the gauge coupling constant gets large at the gaugino condensation scale.

It is interesting to notice, however, that in this type of string models $z = C/|H|^2 \sim (T+\bar{T})e^{(S+\bar{S})/2b_0} \sim 1$ does not necessarily mean that dilaton VEV runs away to $\langle S\rangle \to 0$. We may still obtain dilaton VEV consistent with the weak scale measurement if we allow the compactification radius to be comparable to the gaugino condensation scale, *i.e.* $T \sim R^2 \sim |H|^2 \sim e^{-(S+\bar{S})/2b_0}$. In fact, in a dynamical symmetry breaking scheme proposed in Ref. [3], a compactification radius comparable to the gaugino condensation scale is a requirement in most known string models. In the following, we will briefly review these arguments, and then we will discuss the determination of the dilaton and moduli VEV's in these models.

In Ref [3], we proposed that gaugino condensation may induce the charged background VEV's which breaks gauge symmetry. This dynamics makes possible affine level one grand unified (*i.e.* $SU(5)$ or $SO(10)$) string models with intermediate gauge symmetry breaking scale $M_{\mathrm{GUT}} \sim 10^{16}$ GeV. We show that, in some string models, the quantization condition enforces the compactification radius to be on the order of the gaugino condensation scale. For example, in the orbifold models, the Z_n symmetry requires the Wilson lines to be quantized: $\oint A^i dx_i = 2\pi/n$. For an orbifold with the moduli background fields

$$G^{ij} = R^2\delta^{ij}, \qquad G_{ij} = R^{-2}\delta^{ij}, \tag{24}$$

one has

$$\oint A^i \cdot dx_i = \oint A^i G_{ij} dx^i = \frac{2\pi A}{R} = \frac{2\pi}{n}, \qquad A = \frac{R}{n}. \tag{25}$$

The compactification radius is related to the charged background VEV which is of the order of the gaugino condensation scale.

In the following, we will determine the dilaton and moduli VEV using the above quantization condition and the value for $z = C/|H|^2$ which is dynamically determined through the one-loop effective potential given above. To do this, we use the relation between the compactification radius and the real part of dilaton $\mathrm{Re}(S)$ and moduli VEV $\mathrm{Re}(T)$ [13,1]:

$$R^2 = \mathrm{Re}(S) \cdot \mathrm{Re}(T), \tag{26}$$

we get

$$\mathrm{Re}(S) = \frac{n^2 A^2}{\mathrm{Re}(T)}. \tag{27}$$

The A is related to the H through the tree-level vacuum condition

$$c_0 \equiv d_{ijk} A^i A^j A^k = -\frac{dH^3}{4x_1}. \tag{28}$$

Assuming that all the induced charged background is the same *i.e.* $A_i = A$ and $d_{ijk} \sim 1$, we obtain

$$A = \left(\frac{-1}{4x_1}\right)^{1/3} H. \tag{29}$$

We then obtain

$$Re(S) = n^2 \left(\frac{1}{4x_1}\right)^{2/3} \frac{H^2}{Re(T)} \tag{30}$$

$$= n^2 \left[\frac{1}{2}\left(y + \frac{b_0}{3}\right)\right]^{2/3} \frac{2}{z+1}. \tag{31}$$

It is easy to see that with the dynamically determined z and y, the dilaton VEV can be determined which also depends on the integer n of Z_n symmetry. For example, for E_6 hidden sector, with $y = 1$, $z = 0.73$ and $Re(S) = 0.76n^2$; with $y = 0.1$, $z = 0.22$ and $Re(S) = 0.32n^2$. We see that realistic VEV's for dilaton can be easily obtained. Although the calculation performed here is at best a model of the full gaugino condensation dynamics, the above analysis indicates that the specification of z and the weak scale gauge coupling constant measurement could uniquely determine the n; on the other hand, with the known string model, $Re(S)$ can be dynamically determined.

We conclude that the hierarchy between the moduli VEV and the gaugino condensation scale is not generated with the inclusion of the high-loop corrections to the dilaton Kähler potential. However, the moduli and dilaton VEV's, consistent with weak scale measurements, can be dynamically determined in the models proposed in Ref. [3] by using imposed quantization conditions. A more reliable calculation of dilaton and moduli VEV's in this scenario depends on a better understanding of the gaugino condensation dynamics, since the dilaton mass is on the order of the gaugino condensation scale. We hope the recent progress on the dual descriptions of the dynamics may shed some light on this.

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